Factoring Trinomials

What is a Trinomial?

Trinomials are algebraic expressions that consist of variables, coefficients, and constants that are added, subtracted, and/or multiplied and then raised to a positive exponent. A trinomial can have only three terms and usually is in the form: $x^2 + bx + c$ or $ax^2 + bx + c$

What is Factoring?

Factoring is the process of simplifying an algebraic expression to its simplest products. In this case, we will factor trinomials to their simplest form, which is the product of two binomials.

Four Methods for Factoring Trinomials:

1. **Factoring Trinomials** – Trinomials of the form $ax^2 + bx + c$ can be factored by finding two numbers with a product of $a \cdot c$ and a sum of b, such as $(x + p)(x + q)$ where $p \cdot q = c$ and $p + q = b$. This method is often used when the a of the trinomial has a coefficient of 1, but it can also be used for those with greater coefficients.

Example:		General Format:
$x^2 + 3x - 18$	First, we look at our a and c, which are 1 and −18 respectively, and multiply them.	$ax^2 + bx + c$
Factors of −18 $18 + (-1) = 17$ $(-18) + 1 = -17$ $9 + (-2) = 7$ $(-9) + 2 = -7$ $6 + (-3) = 3$ $(-6) + 3 = -3$	After we multiply them, we will list all the possible factors of the product, −18. Once we list all the factors, we add them and see what factors equal our b, 3.	***Factors of $a \cdot c$*** $\pm p \quad \pm q$ $p + q = b$
$(x + 6)(x + (-3))$	After we find which factors equal our b, we input them in the p and q format.	$(x + p)(x + q)$
$(x + 6)(x - 3)$	Finally, we have found the factored form of our trinomial.	

A quick tip: *If your* b *from the* $x^2 + bx + c$ *form is negative, the larger number, either p or q, will be negative. The same applies if it's positive. If your* b *is positive, the larger number, either p or q, will be positive.*

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2. **Factoring by Grouping** – To factor a polynomial in the form $ax^2 + bx + c$ by grouping, we find two common factors with a product of $a \cdot c$ and a sum of b. With these numbers, we expand bx into the sum of the two terms, and factor each portion of the expression separately, then factor out the GCF of the entire expression. This method is mostly used when the trinomial has an a that is greater than 1, such as $\underline{2}x^2 + 3x + 1$.

Example:		General Format:
$8x^2 - 10x - 3$	Like the previous method, we look at our a and c, 8 and −3, and multiply them.	$ax^2 + bx + c$
Factors of −24 $24 + (-1) = 23$ $(-24) + 1 = -23$ $12 + (-2) = 10$ $\underline{(-12) + 2 = -10}$ $8 + (-3) = 5$ $(-8) + 3 = -5$ $6 + (-4) = 2$ $(-6) + 4 = -2$	Then, we will list all the possible factors of the product −24. Once we list all the factors, we add and see what factors equal our b, −10.	***Factors of $a \cdot c$*** $\pm p \quad \pm q$ $p + q = b$
$8x^2 - 12x + 2x - 3$	Once we find the factors, we will expand b into the sum of the factors.	$ax^2 + px + qx + c$
$8x^2 - 12x \mid + 2x - 3$	Next, we will separate the trinomial into two and view them separately.	$ax^2 + px \mid + qx + c$
$\underline{8x^2 - 12x} \mid + \underline{2x - 3}$	Once separated, we will factor out their greatest common factor (GCF).	$\underline{ax^2} + \underline{px} \mid + \underline{qx} + \underline{c}$
$4x(\underline{2x - 3}) \mid +1(\underline{2x - 3})$	After being factored, the products should be the same for both sides. These products will be a part of the answer. Meanwhile, the GCFs will be combined into the other part.	$ax\,(\underline{x + p}) \mid q(\underline{x + p})$
$(2x - 3)(4x + 1)$	After combining the GCFs and the products, we will have our answer.	$(x + p)(ax + q)$

A quick tip: *Factoring by grouping is commonly used with larger polynomials that exceed three terms.*

3. **Factoring by Borrowing** – To factor a polynomial in the form $ax^2 + bx + c$ by borrowing, we find the product of $a \cdot c$, with the product becoming the new constant, c, and removing the a coefficient to create a new trinomial. This new trinomial will be factored using method #1. Once the trinomial is factored, p and q of the factors will be divided by a.

Example:		General Format:
$3x^2 + 5x + 2$	Like the previous methods, we look at our a and c, 3 and 2, and multiply them.	$ax^2 + bx + c$
$3x^2 + 5x + 2$ $3 \cdot 2 = 6$	Instead of directly finding the factors, we will rearrange the trinomial first.	$ax^2 + bx + c$ $a \cdot c$
$x^2 + 5x + 6$	We will remove our a and replace our initial c with our product of $a \cdot c$ which is 6.	$x^2 + bx + (a \cdot c)$
Factors of 6 $6 + 1 = 7$ $\quad (-6) + (-1) = -7$ $3 + 2 = 5$ $\quad (-3) + (-2) = -5$	Now, we will factor like in method #1. We will list all the possible factors of c. We add the factors and see which equal our b, 5.	***Factors of $a \cdot c$*** $\pm p \quad \pm q$ $p + q = b$
$x^2 + 5x + 6$ $(x + 3)(x + 2)$	After we find which factors equal our b, we input them in the p and q format.	$x^2 + bx + (a \cdot c)$ $(x + p)(x + q)$
$\left(x + \frac{3}{3}\right)\left(x + \frac{2}{3}\right)$	Unlike method #1, we are not done. The last step is to divide p and q by your a.	$\left(x + \frac{p}{a}\right)\left(x + \frac{q}{a}\right)$
$(x + 1)\left(x + \frac{2}{3}\right)$	After some quick simplification, you have the factored form of your trinomial.	

A quick tip: *Factoring by borrowing is used when your trinomial has an* ***a*** *that is greater than 1, like the grouping method. This method is best to use when finding the x-intercepts of a trinomial as it simplifies the process.*

4. **Factoring by Substitution** – To factor a trinomial with substitution, we must substitute expressions—usually the variables—to make the trinomial more manageable when it comes to factoring. In other words, we will substitute certain expressions until the trinomial is in the $ax^2 + bx + c$ form. This method is used when the exponents of a trinomial are greater than 2.

Example:		General Format:
$x^6 - 5x^3 + 4$	First, we will look at the trinomial and identify the substitutable expressions.	$ax^? + bx^? + c$
$x^6 - 5x^3 + 4$ $u = x^3$ $u^2 = x^6$	The substitutable expressions will be the ones with variables, which are x^6 and x^3. We will substitute these as $u = x^3$ and $u^2 = x^6$	$ax^? + bx^? + c$ $u = x^?$ $u^2 = x^?$
$u^2 - 5u + 4$	Once substituted, this will change the trinomial into the $ax^2 + bx + c$ form.	$au^2 + bu + c$
<u>Factors of 4</u> $4 + 1 = 5$ $(-4) + (-1) = -5$ $2 + 2 = 4$ $(-2) + (-2) = -4$	Now we can factor as usual. We list all the possible factors of c, then add the factors and see which equal our b, -5	***<u>Factors of a · c</u>*** $\pm p$ $\pm q$ $p + q = b$
$(u - 4)(u - 1)$	Once we find the correct factors, we input them in the p and q format.	$(u + p)(u + q)$
$u = x^3$ $(x^3 - 4)(x^3 - 1)$	Finally, we replace our u with its original form, x^3, and now we have our answer.	$u = x^?$ $(x^? + p)(x^? + q)$

A quick tip: *Factoring by substitution is used when your trinomial has exponents greater than 2, but it will not always work. For it to work, the exponent for* ax *must be double the exponent for* bx*, such as* $ax^4 + bx^2 + c$ *or* $ax^6 + bx^3 + c$*.*

NOTE: Some Trinomials cannot be factored. If none of the methods work, the trinomial must be solved for its *x*'s by using the quadratic formula. This process converts it into an equation instead of an expression and provides different answers.

References:

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Disclaimer: We did not include all of the resources conferred to formulate this handout. We encourage students to conduct further research to find additional resources. The format of this list is not commensurate with a standard format.