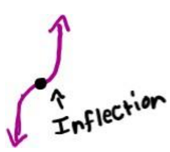
1st and 2nd Derivatives

Key words to consider:

- **Partition Numbers**
  - Solve for the derivative and equal it to zero.
  - Partition numbers include numbers that are in and out of the domain.
  - All critical numbers are partition numbers but not the other way around.
- **Critical Numbers**
  - Solve for the derivative and equal it to zero.
  - Only include numbers in the domain
    - Pay attention to restrictions
    - Does not include numbers that are not define (open circles) or asymptotes
- **Extrema→ Extreme values (Minimums and Maximums)**

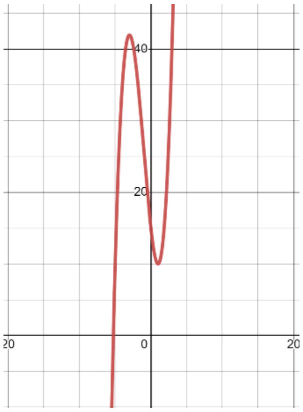
  - Local extrema
    - Any minimum or maximum value in the function
    - Can be more than one value

  - Absolute extrema
    - The highest or lowest value
    - Only one value



  - Inflection point→ where the function changes concavity
    - Concave up
    - Concave down

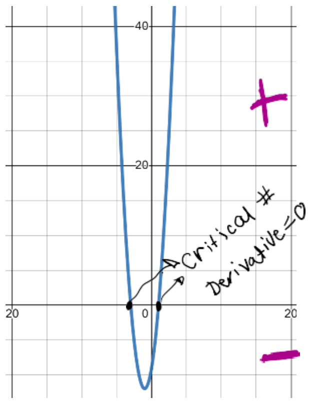
Graphs



Original

$$f(x) = x^3 + 3x^2 - 9x + 15$$

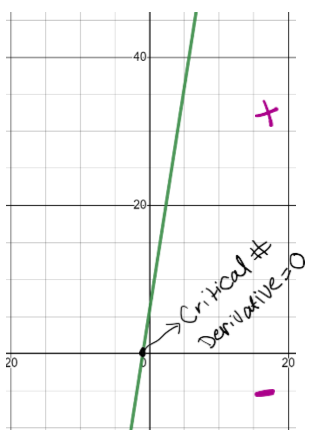
- To find the derivative graphically, we look at the slope of the original function.
  - If the slope is decreasing, then the derivative is negative.
  - If the slope is increasing, then the derivative is positive.



1st Derivative

$$f'(x) = 3x^2 + 6x - 9$$

- To get the Original
  - If below the x-axis, the derivative is negative, and the original function is decreasing.
  - If above the x-axis, the derivative is positive, and the original function is increasing.
- To get the 2nd Derivative, look at the slope.
  - If slope is decreasing, then the 2nd derivative is negative ➔ Concaved down
  - If slope is increasing, then the 2nd derivative is positive➔ Concave up



2nd Derivative

$$f''(x) = 6x + 6$$

- To get the Original
  - If the 2nd derivative is negative, then the original is concave down.
  - If the 2nd derivative is positive, then the original is concave up.

1st Derivative

- The first derivative is the derivative of the original function.
- Used to find local extrema:
  - Minimum
  - Maximum
- Critical numbers tell you where the extrema are.
- Review the completed example below.

$$f(x) = x^3 + 3x^2 - 9x + 15$$

Θ Step 1: Find the Derivative.

$$f'(x) = 3x^2 + 6x - 9$$

Θ Step 2: Equal derivative to zero to get critical numbers.

$$3x^2 + 6x - 9 = 0$$

$$3(x^2 + 2x - 3) = 0$$

$$3(x - 1)(x + 3) = 0$$

Factor
3
1 3

$x - 1 = 0$ $\qquad \boxed{\begin{array}{c} x = 1 \\ x = -3 \end{array}}$ → critical #s

x

Θ Step 3: Do a line chart and choose a number smaller and bigger than the critical number to check if your derivative is positive or negative.

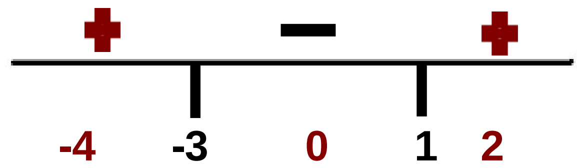
Θ Step 4: Now that you have your test numbers (not the critical numbers), plug those numbers into the derivative.

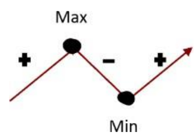
$$f'(x) = 3x^2 + 6x - 9$$

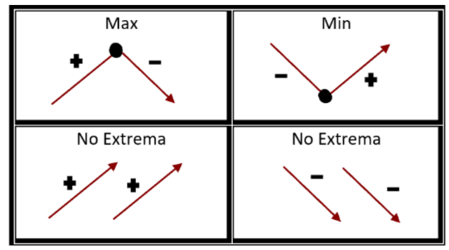
$$f'(-4) = 3(-4)^2 + 6(-4) - 9 = 15$$

$$f'(0) = 3(0)^2 + 6(0) - 9 = -9$$

$$f'(2) = 3(2)^2 + 6(2) - 9 = 15$$







- F(x) has a Max at $x = -3$
  - To get y value plug -3 into the original function
- F(x) has a min at $x = 1$
- F(x) is increasing (+) on the interval $(-\infty, -3) \cup (1, \infty)$
  - It is ∞ and $-\infty$ because we don't know where it ends; the marron values are used to test where the value is positive or negative (increasing or decreasing), not end points.
  - Always check if the function has a restriction ➔ those will be your end points.
- F(x) is decreasing (-) from $(-3,1)$

2nd Derivative

- The second derivative is the derivative of the first derivative.
- Used to find concavity.
  - Concave up
  - Concave down
- Critical numbers tell you where the inflection point is.
- Review the completed example below.

$$f'(x) = 3x^2 + 6x - 9$$

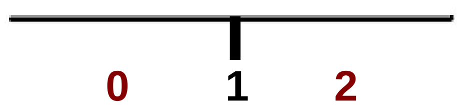
- Step 1: Find the 2nd derivative.

$$f''(x) = 6x + 6$$

- Step 2: Equal the second derivative to zero and solve for x.

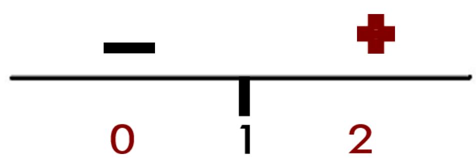
$$6x + 6 = 0$$
$$6x = -6$$
$$x = -1$$ → critical #s

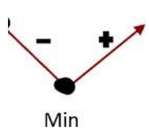
- Step 3: Do a line chart and choose a test number smaller and bigger than the critical number to check if your derivative is positive or negative.

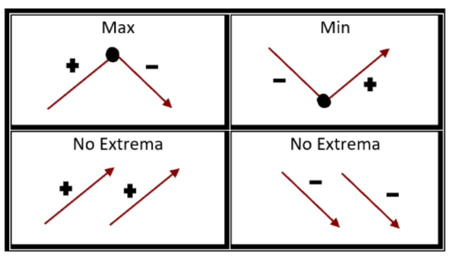


- Step 4: Now that you have your test numbers (not the critical numbers), plug those numbers into the 2nd derivative.

$$f''(x) = 6x + 6$$
$$f'(-2) = 6(-2) + 6 = -6$$
$$f(0) = 6(0) + 6 = 6$$







  - F(x) has an inflection point at $x = -1$
    - To get y value, plug -1 into the original function.
  - F(x) is concaved down from $(-\infty, -1)$
  - F(x) is concaved up from $(-1, \infty)$

Disclaimer: We did not include all of the resources conferred to formulate this handout. We encourage students to conduct further research to find additional resources. The format of this list is not commensurate with a standard format.