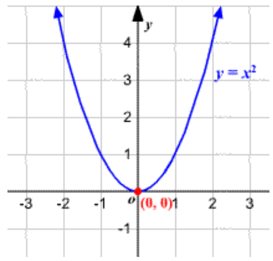
Derivatives Rules

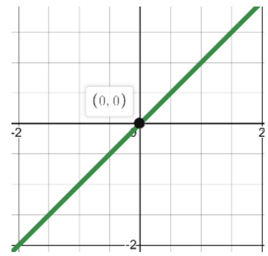
To take a derivative means to find the <u>slope of a line</u> (rate of change) that lies tangent to the function <u>at the specific point</u>.

Key words:

- Slope
- Tangent
- Rate of Change
- Marginal
- Maxima/ Minimum
- Concavity
- $\frac{d}{dx}$

The Derivative is one power down then the original





The following are rules that will assist in finding the derivatives for certain problems.

Extra Rules/ Rewrites

Square Roots	$\sqrt{x} = x^{\frac{1}{2}}$	$\sqrt[3]{x^2} = x^{\frac{2}{3}}$ **2 what you have** **3 what you want**
	$\sqrt[4]{x} = x^{\frac{1}{4}}$	
Fractions I	$\frac{1}{x} = x^{-1}$	***When you add a negative to the exponent it moves up**
	$\frac{25}{x^2} = 25x^{-2}$	***Can be rewritten because the top # is a constant.** ***If it has a variable on the top & bottom, then use Quotient Rule**
Fractions II	$\frac{x^2}{25} = \frac{1}{25}x^2$	$\frac{\mathbf{1}}{\mathbf{5x^2}} = \frac{\mathbf{1}}{\mathbf{5}} * \frac{\mathbf{1}}{\mathbf{x^2}} = \frac{\mathbf{1}}{\mathbf{5}}\mathbf{x^2}$
Zero Exponent	$e^0 = 1$	***Anything to the power of zero equals 1**
	$5^0 = 1$	

e and ln	$e = 2.718\ldots$ $ln = \frac{1}{2.718\ldots}$ $e * ln = 1$	* e by itself is a constant * You can only get the derivative if it has a variable in the exponent *ln is the inverse of e *The derivative of a constant is zero

4 Step Process

Original: $f(x)$	$f(x) = x^2 + 6x - 10$
Step 1: $f(x+\boldsymbol{h})$	$= (x+\boldsymbol{h})^2 + 6(x+\boldsymbol{h}) - 10$ $= (x+\boldsymbol{h})(x+\boldsymbol{h}) + 6(x+\boldsymbol{h}) - 10$ $= x^2 + 2xh + h^2 + 6x + 6h - 10$
Step 2: $f(x+h) - \boldsymbol{f(x)}$	$= x^2 + 2xh + h^2 + 6x + 6h - 10 - \boldsymbol{(x^2 + 6x - 10)}$ $= x^2 + 2xh + h^2 + 6x + 6h - 10 - \boldsymbol{x^2} - \boldsymbol{6x} + \boldsymbol{10}$ $= 2xh + h^2 + 6h$
Step 3: $\frac{f(x+h)-f(x)}{\boldsymbol{h}}$	$= \frac{2xh + h^2 + 6h}{\boldsymbol{h}}$ • You can factor out an h $= \frac{h(2x+h+6)}{\boldsymbol{h}}$ $= 2x + h + 6$
Step 4: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$	$\lim_{h\to 0} 2x + h + 6$ • Substitute h with 0 $= 2x + 0 + 6$
Derivative: $f'(x)$	$\boldsymbol{f'(x) = 2x + 6}$

Derivative Rules

Derivative Rules

Power Rule

1. Separate the terms
2. You multiply the exponent with the front #
3. Subtract a one from the exponent
4. Constants (no variable) will always turn into a zero
5. Put the terms back together and that will be your derivative

$f(x) = x^3 + 5x^2 - 9x + 20$	
x^3	$= (3 * 1)x^{3-1} = 3x^2$
$5x^2$	$= (5 * 2)x^{2-1} = 10x$
$-9x$	$= (-9 * 1)x^{1-1} = -9$
20	$= (20 * 0)x^{0-1} = 0$
$f'(x) = 3x^2 + 10x - 9$	

Product Rule

1. Two functions multiplying
2. Label your functions
   a. It does not matter what letters you use
3. Label your 1st function f(x)
4. Label your 2nd function g(x)
5. Get the derivative of each function
6. Plug in your numbers into

$$f'(x)g(x) + g'(x)f(x)$$

$h(x) = (x^2 + 8)(x^5)$	
$f(x) = x^2 + 8$	$f'(x)g(x) + g'(x)f(x)$
$f'(x) = 2x$	$(2x)(x^5) + (5x^4)(x^2 + 8)$
$g(x) = x^5$	$2x^6 + 5x^6 + 40x^4$
$g'(x) = 5x^4$	$7x^6 + 40x^4$

Quotient Rule

1. Two functions divided
   a. Variable on the top & bottom
2. Label your functions
   a. Order matters (top 1st)/(bottom 2nd)
3. Label top f(x)
4. Label bottom g(x)
5. Get the derivative f(x) & g(x)
6. Plug in your numbers into

$$\frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

$h(x) = \frac{x^2 + 8}{x^5 + 9}$	
$f(x) = x^2 + 8$	$\frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$
$f'(x) = 2x$	$\frac{2x(x^5 + 9) - 5x^4(x^2 + 8)}{(5x^4)^2}$ ☆Distribute
$g(x) = x^5 + 9$	$\frac{2x^6 + 18x - 5x^6 - 40x^4}{(5x^4)^2}$ ☆Combine like terms
$g'(x) = 5x^4$	$\frac{-3x^6 - 40x^4 + 18x}{(5x^4)^2}$

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Chain Rule

Original:	$5(x^2+8)^3$
Step 1: Θ Derivative of the outside Θ What is inside the parenthesis says the same	Outside: $5(x^2+8)^3$ Derivative: $15(x^2+8)^2$
Step 2: Θ Derivative of the inside	Inside: (x^2+8) Derivative: $2x$
Step 3: Θ Multiply Step 1 &2	$15(x^2+8)^2 * 2x$
Derivative:	$30x(x^2+8)^2$

Derivative of ln(x)

Original	**Step 1:** Inverse of the inside	**Step 2:** Derivative of the inside	**Step 3:** Multiply Step 1 & 2	**Derivative**
$\ln(x)$	Inside: x Inverse: $\frac{1}{x}$	Inside: x Derivative: 1	$\frac{1}{x} * 1$	$\frac{1}{x}$
$\ln(2x)$	Inside: $2x$ Inverse: $\frac{1}{2x}$	Inside: $2x$ Derivative: 2	$\frac{1}{2x} * 2$	$\frac{1}{x}$
$\ln(x^2+5)$	Inside: x^2+5 Inverse: $\frac{1}{x^2+5}$	Inside: x^2+5 Derivative: $2x$	$\frac{1}{x^2+5} * 2x$	$\frac{2x}{x^2+5}$
$2\ln(x)$ stays outside	Inside: x Inverse: $\frac{1}{x}$	Inside: x Derivative: 1	$2 * \frac{1}{x} * 1$	$\frac{2}{x}$

Derivative Rules

Derivative of e^x

Original	**Step 1:** The derivative of $e^x = e^x$ (Stays the same)	**Step 2:** Derivative of Exponent	**Step 3:** Multiply Step 1 & 2	**Derivative**
e^x	e^x	Exponent: x Derivative: 1	$e^x * 1$	e^x
$5e^x$	$5e^x$	Exponent: x Derivative:1	$5e^x * 1$	$5e^x$
$2e^{3x}$	$2e^{3x}$	Exponent: $3x$ Derivative: 3	$2e^{3x} * 3$	$6e^{3x}$
$2e^{3x^2}$	$2e^{3x^2}$	Exponent: $3x^2$ Derivative:$6x$	$2e^{3x^2}$*6x	$12xe^{3x^2}$
e^2	No variable= Constant= 0		$0 * 0$	0
$2xe^x$	Has a variable (x) on the outside & on the exponent			Product Rule

Derivative of Exponential

- $\frac{dy}{dx} b^x = b^x \ln(b)$
- $\frac{dy}{dx} 2^x = 2^x \ln(2)$

Derivative of Logarithm

- $\frac{dy}{dx} \log_b x = \frac{1}{\ln(b)} \left(\frac{1}{x}\right)$

Function	**Step 1:** Exponent goes to the front	**Step 2:** $\log_b = \frac{1}{\ln(b)}$	**Step 3:** $\log x = \left(\frac{1}{x}\right)$	**Step 4:** Put all steps together	**Derivative**
$\log_4 x^5$	$5 \log_4 x$	$\frac{1}{\ln(4)}$	$\frac{1}{x}$	$5 * \frac{1}{\ln(4)} * \frac{1}{x}$	$\frac{5}{\ln(4)}\left(\frac{1}{x}\right)$

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Disclaimer: We did not include all of the resources conferred to formulate this handout. We encourage students to conduct further research to find additional resources. The format of this list is not commensurate with a standard format.

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